

Resonant 2D-2D tunneling with account for spin-orbit interaction

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We present a theory of quantum tunneling between 2D layers with account for Rashba and Dresselhaus spin-orbit interaction (SOI) in the layers. Energy and momentum conservation results in a single resonant peak in the tunnel conductance between two 2D layers as has been experimentally observed for two quantum wells (QW) in GaAs/AlGaAs heterostructures. The account for SOI in the layers leads to a complex pattern in the tunneling characteristic with typical features corresponding to SOI energy. For this manifestation of SOI to be observed experimentally the characteristic energy should exceed the resonant broadening related to the particles quantum lifetime in the layers. We perform an accurate analysis of the known experimental data on electron and hole 2D-2D tunneling in AlGaAs/GaAs heterostructures. It appears that for the electron tunneling the manifestation of SOI is difficult to observe, but for the holes tunneling the parameters of the real structures used in the experiments are very close to those required by the resolution criteria. We also consider a new promising candidate for the effect to be observed, that is p-doped SiGe strained heterostructures. The reported parameters of cubic Rashba SOI and quantum lifetime in strained Ge QWs fabricated up to date already match the criteria for observing SOI in 2D-2D heavy holes tunneling. As supported by our calculations small adjustments of the parameters for AlGaAs/GaAs p-type QWs or simply designing a 2D-2D tunneling experiment for SiGe case are very likely to reveal the SOI features in the 2D-2D tunneling.

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I. INTRODUCTION

Quantum tunnelling between two 2D layers separated by a weakly transparent potential barrier is a prime example of resonant phenomena in semiconductor nanostructures. Energy and in-plane momentum conservation put tight restrictions on the 2D-2D tunneling so that the conductance between identical layers exhibits delta function-like maximum at zero bias, while at any finite voltage applied across the barrier the quantum transitions between the layers are forbidden. The phenomena has been observed experimentally for both n-doped and p-doped AlGaAs/GaAs heterostructures with two spatially separated QWs^{1,2}. The resonant peak appearing in the tunneling I-V characteristic at zero bias is broadened to the characteristic width Γ due to scattering in the layers, for QW of a high quality the electron-electron scattering is probably the main contribution³. With account for spin-orbit interaction (SOI) in the 2D layers the tunneling transport turns out to be more complex. SOI splits the size quantization subbands in the layers allowing the resonant tunneling between one spin-orbit subband of the left layer into another subband of the right layer. This can occur when the left and right states has the same energy, i.e. when the spin-orbit splitting is exactly compensated by an external bias. It was demonstrated theoretically that in this way SOI would manifest

itself in the tunnel conductance with two resonant peaks or more complex pattern with typical features shifted from zero bias by a characteristic energy of the spin-orbit splitting^{4,5}. For exactly the same 2D layers the tunneling between opposite spin-orbit subbands (assuming no spin-flip during the tunneling) would be forbidden because of the orthogonality of the spin wavefunctions. However, in real heterostructures with two QWs two doping layers are located below the lowest QW and above the highest, so that the ionized dopant layers create electrical fields which have opposite directions in the two QWs. Consequently, the parameter of Rashba SOI which is proportional to the electric field has the opposite sign in the two QWs so that (in the absence of other spin-orbit contributions) the spin-orbit subbands in one layer are reversed compared to the other one, the spin structure of the upper subband in one layer matches that of the lower subband in the other one. This removes the restriction on the tunneling between the opposite subbands. Introducing SOI of Dresselhaus type (the same in both layers) results in a more complex spin structure of the eigenstates in the layers and more rich tunneling pattern as a result of interference between the two SOI contributions. In our previous works we considered electron tunneling between two n-type 2D layers (n-n tunneling) with account for both Rashba and Dresselhaus contributions and obtained analytical expression for the tunnel-

ing current in this general case^{6,7}. However, the question why the SOI effects had not been seen in the tunneling experiments remained open. For the manifestation of SOI in the 2D-2D tunneling transport to be observed the SOI characteristic energy δ should exceed the resonance broadening Γ . In this paper we generalize the theory and consider the tunneling of heavy holes between two p-type layers (h-h tunneling). We consider parameters of the heterostructures used in the experiments on n-n and h-h tunneling and analyze the requirements for the SOI pattern to be resolved experimentally. Our calculations show that for the p-type AlGaAs/GaAs heterostructures studied experimentally the parameters are very close to the SOI features to be observed in the tunnel current. Moreover, for the p-type case there is a strong scaling with doping level and QW thickness so that a small correction of the parameters must lead to the experimental observation of SOI in the tunneling transport. Another promising candidate for the 2D-2D tunneling experiment is the SiGe strained heterostructures. The strained Ge QWs possess only Rashba SOI, the Dresselhaus term is absent due to lack of bulk inversion asymmetry for Ge lattice. We consider two different heterostructures with p-type QWs studied experimentally^{8,9} for which an accurate data on the SOI interaction was obtained. The SOI in these structures is larger than in AlGaAs case so designing a tunneling experiment is very attractive as our calculations confirm.

II. THEORY

The system under consideration is illustrated by Fig. 1. We consider two QWs separated by a weakly transparent barrier. Keeping in mind the relevant experiments^{1,2} the QWs could be of GaAs grown along [001] direction (z-axis), in-plane axis are $x \parallel [100]$, $y \parallel [010]$, the barriers formed by $\text{Al}_x\text{Ga}_{1-x}\text{As}$. Depending on the doping the QWs can be populated either by electrons or by holes. The first case is shown in Fig. 1(a). Here the QWs for the electrons are formed by the conduction band profile. The doping is made by the two layers of donors as illustrated by red circles. The ionized donors create a positive charge and an electric field directed as shown in the figure. The difference between the chemical potentials of the electrons in the QWs is created by an external bias U applied directly to the QWs which enables the tunneling of the electrons from the right QW to the left QW with electric current flowing in the opposite direction. For the case of h-h tunneling the situation is quite similar as shown in Fig. 1(b). The figure shows the valence band profile and the ionized acceptors form two negatively charged layers so that the direction of the electric field is opposite to that of electron case, the current flows in the same direction as the holes flux. We consider one size quantization level for the electrons for n-n tunneling and one size quantization level for the heavy holes (HH1) in the case of h-h tunneling, zero temperature is assumed.

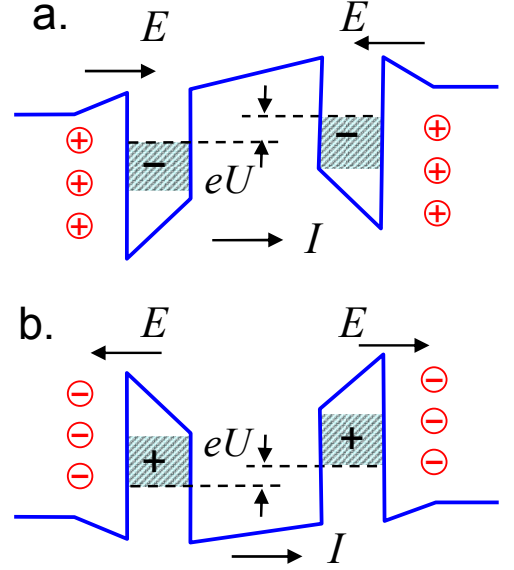


FIG. 1: (Color online) Energy diagram of two 2D layers. The n-n tunneling case (a) is realized by donor impurity doping layers located as illustrated by red circles. The electrons (dashed regions) populate QWs formed by conduction band profile. An external bias U is applied to the QWs which results in the tunneling current I . The electric field E formed by the ionized donor layers has opposite direction in the two QWs. The h-h tunneling case (b) occurs when the doping is p-type. In this case the current between the QWs formed by the valence band potential profile is carried by holes. The electric field in this case E is formed by the ionized acceptor layers with negative charge.

We use the Bardeen's tunneling Hamiltonian approach³ with the total Hamiltonian of the system expressed as:

$$H = H^L + H^R + H_T, \quad (1)$$

where H^L, H^R are the partial Hamiltonians for the left and right layers respectively, H_T is the tunneling term. Assuming spin is conserved during the tunneling the current density is given by⁷:

$$j = \frac{eT^2}{4\pi^3\hbar^3} \text{Re} \left\{ \text{Tr} \int G^R(\mathbf{p}, \varepsilon - eU) G^L(\mathbf{p}, \varepsilon) d\mathbf{p} d\varepsilon \right\}, \quad (2)$$

where ε is the electron energy, \mathbf{p} is its in-plane momentum, $G^{R,L}$ are Green's functions of the left and right layers respectively, T is the Bardeen's tunneling matrix element, the measure of the tunnel coupling between the layers, e is the elementary charge. With electron spin taken into account the Green's functions are 2x2 metrics. The spinor basis states are defined by angular momentum projection on z-axis, for the electrons the basis is $(1/2, -1/2)$ while for the heavy holes it is $(3/2, -3/2)$. The Green's functions for the electrons or holes in the layers are to be calculated with account for SOI and scattering processes in the layers. As long as the interlayer transitions rate is described by the formula (2) the Green's

functions in (2) are aware of intralayer interactions only, any inter-layer scattering processes which could give rise to vertex corrections shouldn't be taken into account in the leading order of the tunneling parameter⁷. We consider two types of SOI Hamiltonians for the electron and holes tunneling respectively. For the n-n tunneling both Rashba and Dresselhaus contributions to SOI in a 2D layer are linear in the in-plane momentum:

$$H_e^{SO} = \alpha_e (k_y \sigma_x - k_x \sigma_y) + \beta_e (k_x \sigma_x - k_y \sigma_y) \quad (3)$$

where k_x, k_y are the in-plane wavevector components, σ_x, σ_y are Pauli matrices. z-axis are normal to the QWs planes, α_e and β_e are the parameters of Rashba and Dresselhaus contributions to SOI. The Rashba term is assumed to be linear in external electric field $\alpha_e \sim E$. For the h-h tunneling between the considered QWs the dominating contribution to the Rashba SOI is cubic in the in-plane wavevector¹⁰, while the Dresselhaus part is linear. We use the following Hamiltonian for the valence band Rashba⁸ and Dresselhaus¹¹ terms:

$$H_h^{SO} = \alpha_h i (k_-^3 \sigma_+ - k_+^3 \sigma_-) + \beta_h (\sigma_x k_x + \sigma_y k_y), \quad (4)$$

where $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$, $k_{\pm} = k_x \pm ik_y$. With no other interactions taken into account the Hamiltonian for a 2D layer takes the form

$$\hat{H} = -\frac{\hbar^2}{2m} \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} \right) + \hat{H}^{SO}, \quad (5)$$

where m is the in-plane mass for the relevant carriers. Its eigenvalues ε and eigenvectors ϕ give the two spin-orbit subbands:

$$\varepsilon_{\mp} = \frac{\hbar^2 k^2}{2m} \pm |u|, \quad \phi_{\mp} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ \gamma^* \end{pmatrix}, \quad (6)$$

where for the electrons

$$u = u_e \equiv \alpha_e i k_- + \beta_e k_+,$$

for the heavy holes

$$u = u_h \equiv \alpha_h i k_-^3 + \beta_h k_-,$$

for both cases $\gamma = u/|u|$. We will now assume that intralayer scattering processes do not involve spin so that the only spin-dependent interaction is the SOI described by (3) or (4). Therefore, in the basis of the eigenstates (ϕ_-, ϕ_+) the single-particle Green's function of the carrier in a 2D layer is expressed as:

$$G = \begin{bmatrix} G_- & 0 \\ 0 & G_+ \end{bmatrix},$$

where

$$G_{\pm}(k, \varepsilon) = \frac{1}{\varepsilon + \varepsilon_F - \frac{\hbar^2 k^2}{2m} \pm |u| + i \frac{\hbar}{2\tau} \text{sgn } \varepsilon}, \quad (7)$$

ε_F being the Fermi level of the layer in the absence of applied voltage. The quantum lifetime τ incorporates all the scattering processes in the layer. It was suggested that in the experiments on 2D-2D resonant tunneling in AlGaAs based heterostructures^{1,2} the key contribution to a particle quantum lifetime τ comes from the e-e or h-h Coulomb scattering. In the standard spinor basis the same Green's function matrix is:

$$G = \frac{1}{2} \begin{bmatrix} G_- + G_+ & \gamma(G_- - G_+) \\ \gamma^*(G_- - G_+) & G_- + G_+ \end{bmatrix}. \quad (8)$$

Inserting (8) into (2) and assuming the spin-orbit splitting small compared to the Fermi energy we end up with the expression for the tunneling current density:

$$j = \frac{emT^2}{4\pi^2\hbar^3} \times \int_0^{2\pi} \left[\left(\frac{eU\Gamma}{(eU - \xi_-)^2 + \Gamma^2} + \frac{eU\Gamma}{(eU + \xi_-)^2 + \Gamma^2} \right) (1 + \text{Re}\gamma^L \gamma^{R*}) \right. \\ \left. + \left(\frac{eU\Gamma}{(eU - \xi_+)^2 + \Gamma^2} + \frac{eU\Gamma}{(eU + \xi_+)^2 + \Gamma^2} \right) (1 - \text{Re}\gamma^L \gamma^{R*}) \right] d\varphi \quad (9)$$

where

$$\Gamma = \frac{\hbar}{\tau} \\ \gamma^{L,R} = \frac{u^{L,R}}{|u^{L,R}|} \\ \xi_{\pm} = |u^R(k_F, \varphi)| \pm |u^L(k_F, \varphi)|, \quad (10)$$

indices L and R denote left and right layer, respectfully, φ is the polar angle for the in-plane wavevector $\mathbf{k} = (k, \varphi)$, $k_F = \sqrt{2m\varepsilon_F}/\hbar$. In the case of no SOI or if it is identical in the two layers, i.e. $u^L = u^R$ (9) reduces to the well-known result³:

$$j = \frac{2emT^2\Gamma}{\pi\hbar^3} \frac{eU}{(eU)^2 + \Gamma^2}. \quad (11)$$

Another limiting case appears when we take Rashba terms in the two layers being of the same magnitude but of the opposite sign and completely neglect Dresselhaus terms^{5,6}. That is

$$j = \frac{emT^2\Gamma}{\pi\hbar^3} \left(\frac{eU}{(eU - 2\delta)^2 + \Gamma^2} + \frac{eU}{(eU + 2\delta)^2 + \Gamma^2} \right), \quad (12)$$

where

$$\delta = \alpha_e k_F \quad (\text{for electrons}) \\ \delta = \alpha_h k_F^3 \quad (\text{for holes}).$$

Instead of a single resonance at zero bias in the absence or identical SOI (11) now we have two resonances located at 2δ . Adding a finite Dresselhaus contribution (equal in

both layers) makes the picture more complex as a result of interference between the spin structure of left and right layers states. Typical resonant features in the tunneling I-V characteristic still occur at a bias corresponding to Rashba energy δ_R given by (13) and Dresselhaus characteristic energy, $\delta_D = \beta_{e,h} k_F$. The so-called 'spin helix' case¹² corresponds to equally strong Rashba and Dresselhaus contributions, that is

$$\begin{aligned}\beta_e &= |\alpha_e| & (\text{for electrons}) \\ \beta_h &= |\alpha_h| k_F^2 & (\text{for holes})\end{aligned}$$

The current density in this case is given by:

$$\begin{aligned}j &= \frac{emT^2 eU}{\pi \hbar^3} \left(\frac{|\text{Im}\sqrt{g_-}|}{|g_-|} + \frac{|\text{Im}\sqrt{g_+}|}{|g_+|} \right) \\ g_{\pm} &= (eU - i\Gamma)^2 \pm 8\delta^2 \\ \delta &= 2\beta_{e,h} k_F.\end{aligned}\quad (13)$$

The resonant features in this case appear at $eU = \pm 2\sqrt{2}\delta$ with rather unusual behaviour, a decrease in Γ suppresses the current⁶.

From experimental point of view the complex resonant pattern in the I-V curve would be resolved only if the characteristic SOI energy δ exceeds the broadening Γ . As will be shown below this is not the case for the existing experiments on n-n¹ and h-h² tunneling which explains why the pattern related to SOI was not revealed. However, for the h-h tunneling in particular, only slight adjustment of the structure parameters would immediately lead to a well resolved fine structure of I-V characteristic related to SOI.

III. ESTIMATES AND CALCULATIONS

A. AlGaAs/GaAs heterostructures

Let us review the relevant parameters affecting the position of SOI related resonant peaks on I-V curve and the broadening in a real experimental situation. Everywhere below we assume Rashba parameters having opposite sign in the two layers due to the opposite direction of the electric field as illustrated by Fig. 1, that is $\alpha_{e,h}^L = -\alpha_{e,h}^R$. Dresselhaus SOI parameters are considered the same for both layers $\beta_{e,h}^L = \beta_{e,h}^R$. For the n-n tunneling we rely on the SOI parameters reported for AlGaAs/GaAs heterostructures with a QW of 12 nm thickness and n-type Si doping in the barrier¹² as these structures are rather similar to those used in the n-n tunneling experiment¹. The Rashba parameter and the sheet electron density in these samples are listed below:

$$\begin{aligned}\alpha_e &\approx 2 \cdot 10^{-11} \text{ eV} \cdot \text{cm} \\ n &\approx 5 \cdot 10^{11} \text{ cm}^{-2}\end{aligned}\quad (14)$$

Parameter	Electrons (n-n)	Holes (h-h)
Sheet density	$1.6 \cdot 10^{11} \text{ cm}^{-2}$	$7.2 \cdot 10^{10} \text{ cm}^{-2}$
Rashba parameter $\alpha_{e,h}$	$6 \cdot 10^{-12} \text{ eV} \cdot \text{cm}$	$1.3 \cdot 10^{-22} \text{ eV} \cdot \text{cm}^3$
Dresselhaus parameter $\beta_{e,h}$	$3 \cdot 10^{-11} \text{ eV} \cdot \text{cm}$	$3.6 \cdot 10^{-10} \text{ eV} \cdot \text{cm}$
Rashba energy δ_R	0.005 meV	0.05 meV
Dresselhaus energy δ_D	0.03 meV	0.25 meV
Broadening $\Gamma = \hbar/\tau$	0.17 meV	0.17 meV

TABLE I: List of parameters relevant to the experiments on n-n tunneling¹ and h-h tunneling² between two QWs in AlGaAs/GaAs based heterostructure.

The electric field created by the charged plane of the ionized donors layer (Fig. 1(a)) can be estimated as (SI units):

$$E = \frac{en}{2\varepsilon\varepsilon_0}, \quad (15)$$

where ε is the dielectric constant of the QW material, e is the elementary charge, ε_0 is the vacuum permittivity. With $\varepsilon = 12$ for GaAs at low temperature from (14) and (15) we get $E \approx 4 \cdot 10^4 \text{ V/cm}$. The Rashba coefficient α_e is proportional to the electric field $\alpha_e = r_e E$, from the values above we get:

$$r_e \approx 5 \cdot 10^{-16} \text{ e} \cdot \text{cm}^2. \quad (16)$$

We further apply this value to the experimental data on 2D n-n tunneling¹. Knowing the sheet electron density we obtain the value for Rashba parameter and Rashba spin-orbit energy $\delta_R = \alpha_e k_F$. For an estimate of Dresselhaus parameter β_e we use the value obtained in the same experiment¹² as is and neglect its obviously weaker dependence on the QW thickness, the corresponding Dresselhaus characteristic energy is $\delta_D = \beta_e k_F$. The values used for the analysis of the n-b tunneling experiment are summarized in Table I together with the quantum lifetime measured as the resonance broadening $\Gamma = \hbar/\tau$ in Ref.¹. As can be seen from Table I, for the n-n tunneling experiment¹ the resonance broadening Γ substantially exceeds the spin-orbit energies δ_R, δ_D , so that the SOI features could not be observed. Fig. 2 shows the normalized differential conductivity calculated using (9) for the n-n tunneling with the parameter set listed in Table I (red curve). There's no visible difference from the case of the tunneling with no SOI at all. Let us now address the question how the parameters are to be changed for the SOI to be revealed. Note that the Rashba energy δ_R is proportional to the electric field and Fermi wavevector k_F , so it scales with the sheet electron density as $\delta_R \sim n^{3/2}$, the scaling for Dresselhaus SOI is $\delta_D \sim n^{1/2}$. We will assume Γ independent of n (if we considered the electron-electron scattering to dominate, its scaling would be¹³ $\Gamma \sim 1/n$, however other scattering processes could start playing a role with sheet density varied). Thus to make the SOI larger than Γ the sheet electron density must be increased by almost an order of magnitude from $n \sim 10^{11} \text{ cm}^{-2}$ to $n \sim 10^{12} \text{ cm}^{-2}$.

Blue curve in Fig. 2 shows the calculation for this particular case. The SOI-related resonances are rather well-resolved. Whether the required substantial increase of the doping level can be realized technologically without affecting other structure parameters (including the dominating of e-e scattering) is not that clear but the task does not seem completely unrealistic. As we will show below the situation for the holes is far more favorable.

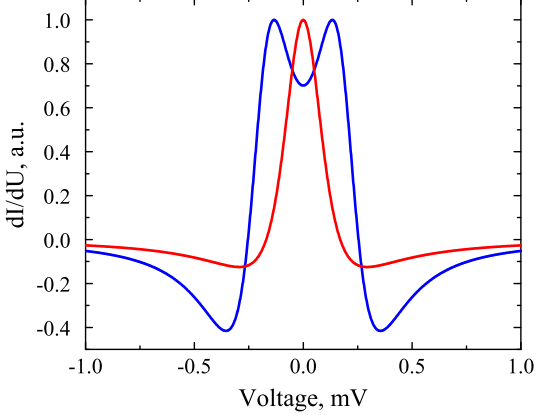


FIG. 2: (Color online) The calculated tunneling differential conductivity for n-n tunneling case. Red curve represents calculation with the parameters relevant to the experiment¹ listed in Table I, blue curve corresponds to the sheet electron density being increased up to $n \sim 10^{12} \text{ cm}^{-2}$.

For the case of h-h tunneling we take an estimate for $r_h = \alpha_h/E$ from Ref.¹⁴ for 20 nm thick QW. That is

$$r_h(20 \text{ nm}) \approx 7.5 \cdot 10^{-26} \text{ e} \cdot \text{cm}^4 \quad (17)$$

Unlike electron case the Rashba SOI for the heavy holes depends on the separation between LH and HH subbands in the QW, therefore it is supposed to be strongly dependent on the QW thickness a . Assuming infinitely deep QW this dependence is $r_h \sim 1/a^4$ (valid while the perturbation theory holds, i.e. the SO splitting energy is smaller than the LH-HH level separations)^{10,14}. Thus, for a narrower 15 nm thick QW used in h-h tunneling experiments we obtain:

$$r_h(15 \text{ nm}) = r_h(20 \text{ nm}) \left(\frac{15}{20} \right)^4 \approx 2.4 \cdot 10^{-26} \text{ e} \cdot \text{cm}^4 \quad (18)$$

An estimate for Dresselhaus SOI contribution for the HH1 subband in a GaAs QW grown along [001] appears to be less reliable as there is no clear experimental data here. From the theoretical calculations with account for QW interface mixing contribution the Dresselhaus parameter for the holes HH1 level in a 10 nm thick GaAs [001] QW can be estimated as¹¹ $\beta_h \approx 3.6 \cdot 10^{-10} \text{ eV} \cdot \text{cm}$. With the sheet hole density reported for the experiment² $p = 7.2 \cdot 10^{10} \text{ cm}^{-2}$ and the corresponding Fermi wavevector $k_F \approx 7.4 \cdot 10^5 \text{ cm}^{-1}$ we get the values summarized in Table I for the h-h tunneling experiment². Note that

the spin-orbit energies δ_R, δ_D are by an order of magnitude higher than those for n-n tunneling case and they are of the same order as Γ . Fig. 3 shows the calculated differential conductivity for the h-h tunneling with the parameters listed in Table I (solid curve). Dotted

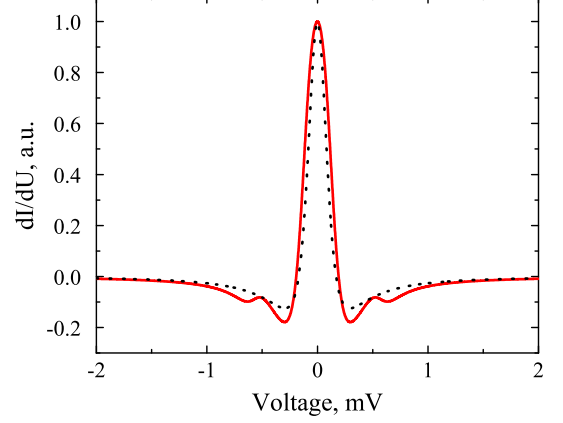


FIG. 3: (Color online) The calculated tunneling differential conductivity for h-h tunneling case. Red curve represents calculation with the parameters relevant to the experiment² listed in Table I, the dotted curve shows the case of no SOI in the layers.

curve in Figs. 3,4 shows the calculated conductivity with no account for SOI as a reference. Thus, the characteristic shown in Fig. 3 is supposed to reproduce the experimental result reported in Ref.². The SOI features are almost not resolved, although a small oscillating feature at $U \approx \pm 0.3 \text{ mV}$ is surprisingly similar to the one seen in the experimental curve at -0.3 mV bias in². Anyway, unlike the electron case the set of parameters is very close to that required for experimental observation of SOI in the h-h tunneling. Moreover, because the main contribution to Rashba SOI for the heavy holes is cubic in the wavevector, the scaling on the sheet hole density p is stronger than for the electrons. With the same arguments as above we have now $\delta_R \sim p^{5/2}$. Increasing p by a factor of 2 would make both δ_R, δ_D far larger than Γ so that all the details including the interference between Rashba and Dresselhaus contributions would be clearly resolved. This is demonstrated by calculation result shown in Fig. 4. Also due to the strong dependence of α_h on the QW thickness a similar enhancement in δ_R would be achieved if the QW width thickness was increased from 15 nm up to 20 nm. The Dresselhaus parameter β_h is also known to depend on the QW thickness¹¹, however not that strongly as the Rashba term, we neglect this dependence in the calculations for the 20 nm thick QW also shown in Fig. 4. Note that the patterns in Fig. 4 are somewhat different. Changing the QW thickness does not affect the Dresselhaus energy δ_D while changing the sheet density affects δ_D through k_F .

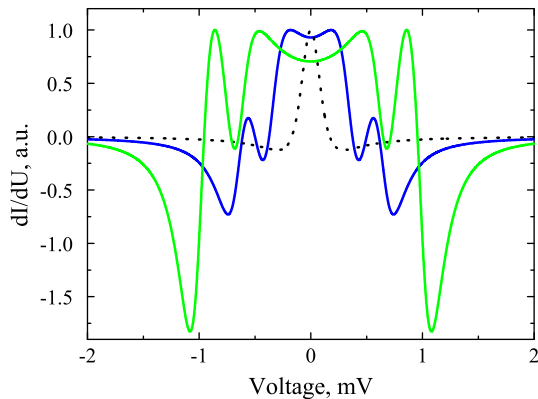


FIG. 4: (Color online) The calculated tunneling differential conductivity for h-h tunneling case with sheet hole density increased by a factor of 2 (green curve) up to $p = 1.4 \cdot 10^{11} \text{ cm}^{-2}$ and for the QW thickness increased up to $a = 20 \text{ nm}$ (blue curve), the dotted curve shows the case of no SOI in the layers.

B. Ge/SiGe heterostructures

Another promising material for the SOI to be observed in the 2D-2D tunneling is a SiGe based heterostructure. Having a central-symmetric lattice Ge semiconductor do not have Dresselhaus SOI, thus for Ge QWs the only contribution to SOI is the Rashba term. Similarly to the AlGaAs/GaAs case discussed above SOI for the heavy holes is larger than for the electrons making h-h tunneling experiment more attractive. Recently strained p-type Ge QWs were fabricated having a large HH-LH subbands separation which results in a strong domination of cubic-Rashba SOI^{8,9}. In the experiment⁸ the SOI splitting $\delta_h = \alpha_h k_F^3$ was reported to be $0.3 - 0.4 \text{ meV}$. The corresponding value of $r_h = 2.3 \cdot 10^{-27} e \cdot \text{cm}^4$, is an order of magnitude smaller than for GaAs QW (18) which is the direct consequence of the LH-HH separation being larger than in GaAs QW. However, due to higher sheet density and additional external electric field Rashba energy in these structures is nearly an order of magnitude higher than that listed in Table I for h-h tunneling in GaAs QWs. Unfortunately, the gain in δ_h appears to be less than the loss in the broadening Γ . The transport time in the QWs studied in Ref.⁸ was $\tau_{tr} \sim 0.2 \text{ ps}$ which gives a lower bound for $\Gamma > \hbar/\tau_{tr} \sim 3 \text{ meV}$. At that δ_h/Γ is well below 1 so the SOI-related structure in 2D-2D tunneling cannot be resolved. An example of a typical set of parameters from Ref.⁸ is listed in Table II (Experiment 1). On the contrary, the Ge QWs studied in Ref.⁹ are already suitable for the tunneling experiment. The difference from⁸ is in smaller LH-HH separation (for the same QW thickness) due to optimized strain which leads to a strong enhancement of α_h . Fig. 5 presents the calculated tunnel characteristic using the parameters of 'SiGe1' sample from Ref.⁹ listed in Table II (Experiment 2). As there is no Dresselhaus term the current density

Parameter	Experiment 1	Experiment 2
Rashba parameter α_h	$1.6 \cdot 10^{-23} \text{ eV} \cdot \text{cm}$	$5 \cdot 10^{-23} \text{ eV} \cdot \text{cm}^3$
Rashba energy δ_R	0.35 meV	1.6 meV
Broadening $\Gamma = \hbar/\tau$	$> 3.7 \text{ meV}$	1.3 meV

TABLE II: Rashba parameters and resonance broadening for strained Ge QWs studied in Ref.⁸ (Experiment 1) and in Ref.⁹ (Experiment 2). QW thickness is $a \approx 11 \text{ nm}$ in both cases.

follows the reduced expression (12). As seen from the fig-

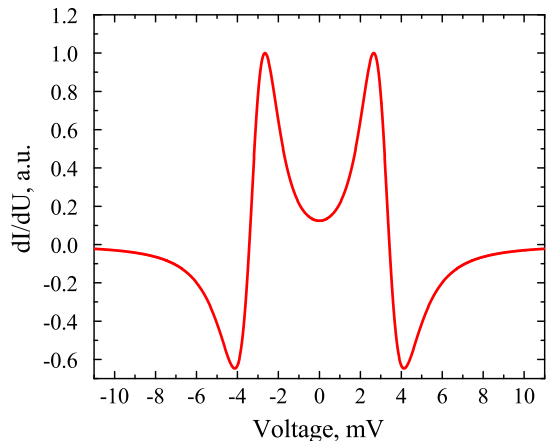


FIG. 5: (Color online) The calculated tunneling differential conductivity for h-h tunneling case using the parameters of the strained Ge QW studied in⁹ listed in Table II (Experiment 2).

ure the two resonances at $\pm 2\delta$ are clearly resolved, thus a 2D-2D tunneling experiment using the two strained Ge QWs is very likely to reveal this SOI pattern.

IV. SUMMARY

The tunneling between 2D layers with SOI separated by a potential barrier has a nontrivial resonant character that reflects the spin structure of eigenstates in the layers. Consequently, the dependence of tunneling conductance on the voltage applied across the barrier appears to be very sensitive to the parameters of SOI. At the same time the homogeneous broadening of the resonant peaks due to a finite quantum lifetime of the particles in the layers smears out the interference pattern. This trade-off between the characteristic spin-orbit splitting energy and the inverse quantum lifetime is crucial for experiment. We have carefully examined the parameters of the reported experiments on 2D-2D resonant tunneling in AlGaAs/GaAs heterostructures and calculated the tunnel current using the developed theory. The calculation confirms that the SOI interference pattern is unlikely to be resolved with the given parameters set. However, a feasible adjustment of the sheet density or the QW thickness would change the situation radically. Of particular

interest is the case of the tunneling between heavy holes subbands. The SOI energy in this case is by an order of magnitude higher than for the n-n tunneling. So as we have shown increasing the sheet hole density in the QWs by a factor of 2 or increasing the QWs thickness from 15 nm to 20 nm would immediately lead to a well-pronounced fine structure of the I-V characteristic related to SOI in the QWs. Another very promising candidate for the suggested 2D-2D tunneling experiment is the Ge strained QWs with exclusively Rashba SOI. We considered parameters of two different heterostructures studied experimentally. The calculation of 2D-2D tunneling using the parameters of the QWs experimentally studied in

Ref.⁹ shows well-resolved fine structure, which is likely to be observed in the experiment. We believe that such a tunneling experiment definitely is worth considering.

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